magnitude of the coefficients. However, when the average mass transfer coefficient was matched to the total drop formation time, a linear correlation was again obtained on semilogarithmic coordinates. Figure 2 shows this result for the present study of dispersed phase resistance.

DISCUSSION

Mathematical models for mass transfer during drop formation, summarized by Popovich et al. (1964) and more recently by Walia and Vir (1976), are generally based on a solution to the diffusion equation in the continuous phase and do not account for circulation within the forming drop. Zimmermann et al. (1980) report short formation time mass transfer results strongly influenced by internal convection. A study of water extraction by isobutanol drops (Heertjes et al., 1954) showed marked circulation for rapid formations, typical drops having a Reynolds number on the order of 0.008. Drop Reynolds numbers in the present project were in the range from 0.05 to 0.5; it therefore seems reasonable to expect rapid circulation in all these drops and a strong dependence on formation time.

The most interesting observation of this study would appear to be the correlation obtainable between average mass transfer coefficient and total drop formation time. This is evident for systems controlled both by the continuous phase resistance (Figure 1) and by the dispersed phase resistance (Figure 2). The lower slope of the line in Figure 1 is consistent with the expectation that internal movement of the drop should not be as significant in determining mass transfer rates at the interface when the resistance lies in the continuous phase. Nevertheless, shear transfers across the interface and generates convection in the continuous phase as well. Thus it appears that total formation time may be the most important parameter in setting the mass transfer coefficient during generation of a dispersed phase.

LITERATURE CITED

Heertjes, P. M., W. A. Holve, and H. Talsma, "Mass Transfer Between Isobutanol and Water in a Spray-Column," *Chem. Eng. Sci.*, **3**, 122 (1954).

Humphrey, J. A. C., R. L. Hummel, and J. W. Smith, "Note on the Mass Transfer Enhancement Due to Circulation in Growing Drops," *Chem. Eng. Sci.*, **29**, 1496 (1974).

Popovich, A. T., R. E. Jervis, and O. Trass, "Mass Transfer During Single Drop Formation," *Chem. Eng. Sci.*, **19**, 357 (1964).

Rajan, S. M., and W. J. Heideger, "Drop Formation Mass Transfer," *AIChE J.*, **17**, 202 (1971).

Walia, D. S., and D. Vir, "Interphase Mass Transfer During Drop or Bubble Formation," *Chem. Eng. Sci.*, **31**, 525 (1976).

Zimmermann, V., W. Halwachs, and K. Schugerl, "Mass Transfer Investigations During Droplet Formation by Means of a Modified Liquid Scintillation Technique," *Chem. Eng. Commun.*, **7**, 95 (1980).

Controller Tuning Using Optimization to Meet Multiple Closed-Loop Criteria

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INTRODUCTION

The PID controller family and its variations continue to be widely used in the process industries, both in analog and digital applications. Tuning or controller design techniques include the methods of Ziegler and Nichols (1942), Cohen and Coon (1953), Harriott (1964), Chidambara (1970), Lopez et al. (1969), and Yuwana and Seborg (1982).

Although the available controller design relations are convenient and easy to use once a simple model is known, they do not really yield standard system performance results from case to case. In particular, the resonant peaks and gain margins vary widely, and in some cases the controllers may be quite unsatisfactory—the response to a disturbance would be much too oscillatory. With some techniques, e.g., Ziegler-Nichols, an unstable system occasionally is obtained. Graphical methods, such as the one recently presented by Edgar et al. (1981) based on the interactive use of a graphics terminal, yield much superior results in terms of system performance, but can be quite time consuming to use.

In general, tuning methods that are based on just one particular input signal or on one particular closed-loop characteristic, such as the decay ratio, have been criticized. This paper suggests a new technique for controller tuning based on several characteristics of the closed-loop process and not on any particular type of input signal. Bollinger et al. (1979) have taken a related approach in the control of a synchronous generator using a lag-lead element. They use the method of inequalities (with moving boundaries) presented by Zakian and Al-Naib (1973). Several functionals describing the desired system behavior in terms of inequalities are defined. Controller settings are then found that meet the inequalities. Zakian and Al-Naib (1973) perform a similar design for a PI controller. The results are superior to standard design methods; however, they may not be consistent from case to case since inequalities are used.

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Substantial user interaction is required in resetting the inequalities (moving boundaries) to improve performance, and they suggest that the use of a graphics terminal would be advantageous.

The technique presented here is based on the use of a single comprehensive index of performance rather than on a set of inequalities. One goal is to obtain consistent closed-loop performance with minimal user expertise (and thus interaction) required. The class of systems examined is that of linearizable single-input, single-output systems.

ALGORITHM DEVELOPMENT

The basis of this tuning method is rooted in the definition of an appropriate index of performance (IP). The approach taken is to characterize the controlled process in terms of its closed-loop frequency domain representation. Then any combination of frequency domain criteria can be chosen as characteristics of the desired controlled process. In this work selection of the resonant peak ratio (M_r), phase margin (PM), and a maximum resonant frequency (ω_r), subject to the former requirements, was found to give the best results. A moderate M_r tends to yield a system that is not too oscillatory, a properly selected PM ensures stability, and a maximum ω_r gives a fast response. (Special desired features, such as a certain decay ratio, also can be accommodated within this IP, as is explained below.) An IP that embodies these characteristics can be defined as

$$\mathrm{IP} = w_m|M_r - M_d|/M_d + w_f/\omega_r + |\mathrm{PM} - \mathrm{PM}_d|/\mathrm{PM}_d \quad (1)$$

where subscripts r and d denote "at ω_r" and "desired value at ω_r" and w_f and w_m are constant relative weighting factors associated with the resonant frequency and the normalized peak ratio at the resonant frequency. Values for these weighting factors were selected so that the three terms in the IP were of the same order. Reasonable values (used in all cases) for w_f and w_m were found to be 0.5 and 10.0, respectively. There is no reason to suspect that these values would need to be changed for any other case. By minimizing this form of IP, the computed values of M_r and PM will be forced close to the desired values, while ω_r will be made as large as possible. These are traditional control system design objectives.

For an assumed form of controller, the minimization is carried out with respect to the settings of the controller parameters. In this work, the form of the controller was taken as PI or PID; if PID, a ratio of 4 between the integral and derivative time constants was taken. This ratio was selected because it is fairly common; for example, it was used by Ziegler and Nichols. Any other ratio could be used. (Allowing τ_D to be independent of τ_I gave no better results.) Thus, in either case, two parameters, K_c and τ_I, had to be found that minimized the IP. To accomplish this minimization, a model of the process is needed so that each possible set of controller settings can be evaluated with respect to the IP by the optimization scheme. Any linear process model can be used, for example, a high-order model, if available. If not, an approximate model can be used, such as second-order with dead time. The model can be obtained in any convenient manner.

Since only numerical values of the IP are available, a direct optimization scheme must be employed. The complex method presented by Box (1965) (a version of the simplex method) was used, but any similar method would suffice. As part of the IP calculation, ω_r, M_r, and PM must be determined for each possible pair of controller settings. This was accomplished using a quadratic fit technique described by Luenberger (1973). Again, other techniques could be used. Hence, for each evaluation of the IP two searches must be made: for the resonant frequency of the closed-loop transfer function and for the frequency that yields unity magnitude for the open-loop transfer function (the defining relationship for the PM). Appropriate formulations for these "inner loop" searches are

$$\mathrm{IP}_a = \min_{\omega} \left[-|\mathrm{CLTF}(\omega)|\right] \quad (2a)$$

$$\mathrm{IP}_b = \min_{\omega} \left[\left|1 - |\mathrm{OLTF}(\omega)|\right|\right] \quad (2b)$$

TABLE 1. PROCESSES STUDIED

Process (Order)	K_p	τ_d	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
2 DT-1	1	0.5	2	1	—	—	—	—
2 DT-2	1	1	10	5	—	—	—	—
3	1	0	1	1	1	—	—	—
4	1	0	2	1	0.2	0.1	—	—
6	1	0	2	1	1	0.2	0.1	0.1

Then ω_r is the frequency minimizing Eq. 2a, M_r is the normalized magnitude of the closed-loop transfer function evaluated at that frequency, and the PM is the phase angle of the open-loop transfer function evaluated at the frequency minimizing Eq. 2b plus 180°. The gain margin (GM) can be found similarly by determining the frequency yielding an open-loop phase angle of −180°.

Finally, values for the desired M_r and PM must be selected. An M_r between 1.1 and 1.4 is a standard criterion, so a value of 1.3 was selected. Standard choices for the PM are 30° and 45°. A conservative value of 45° was chosen. In addition, the IP was severely penalized if the PM fell below 30° or was over 55° and if the GM fell below 1.8.

EXAMPLES AND DISCUSSION

Controllers were found for the processes listed in Table 1 using the IP defined by Eq. 1. Other formulations of the IP gave no better results. The actual process model was used in formulating the algorithm, and, in the cases involving higher-order processes, approximate second-order with dead-time models were also tested.

The controllers obtained and the characteristics of the resulting systems are given in Tables 2, 3, 4, and 5. In all cases the M_r, ω_r, PM, and GM shown in the tables were found using the controller with the actual process model, since the controller would be used with the actual process. The resulting controllers and system characteristics can be compared to corresponding values for the Ziegler-Nichols (ZN) controllers shown in the same tables.

Table 2 shows the results for the two second-order with dead-time processes. The ZN technique gave satisfactory controllers for these processes. However, the systems designed using the new method are much more consistent with respect to M_r and PM. The PM is within 10% of 45°, and M_r is 1.30 in all cases. The second example presented in Table 2 is a process for which Edgar et al. (1981) also designed controllers. Both methods gave comparable results in terms of closed-loop system performance. Figure 1 shows the response to a unit step change in set point for the resulting closed-loop systems obtained using these two PID controllers as well as the ZN PID controller. The responses of the two systems obtained by computer are quite similar, whereas the ZN response is much more oscillatory.

Table 3 shows the results for the third-order process. The controllers tuned using the actual model give the desired closed-loop results and are superior to the ZN results, particularly for the PI

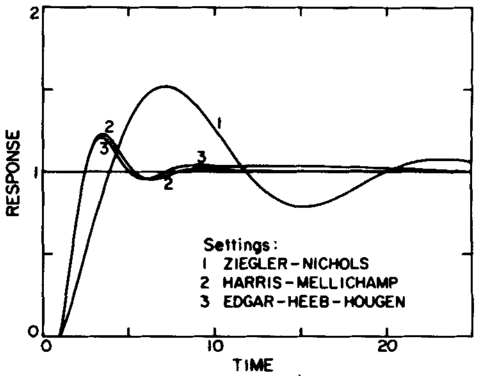

Figure 1. Closed-loop response of process 2DT-2 to a unit stepchange in set point using PID control.

TABLE 2. RESULTS FOR SECOND-ORDER WITH DEAD-TIME PROCESSES

Process	Mode	Tuning Method*	K_c	τ_I	τ_D	M_r	ω_r	PM	GM
2 DT-1	PI	HM	2.00	3.80	—	1.30	0.889	49.2	2.77
		ZN	3.04	3.14	—	2.68	1.14	24.9	1.73
	PID	HM	4.30	2.52	0.629	1.14	1.74	46.7	2.28
		ZN	4.05	1.89	0.472	1.78	1.50	27.4	2.11
2 DT-2	PI	HM	3.32	25.73	—	1.30	0.236	46.5	4.12
		EHH	2.92	20.0	—	1.25	0.21	47.9	4.48
		ZN	7.08	9.72	—	6.69	0.361	9.13	1.43
	PID	HM	10.70	13.56	3.39	1.30	1.00	49.0	2.17
		EHH	9.44	9.0	4.0	1.28	1.07	51.6	2.14
		ZN	9.44	5.83	1.46	2.24	0.385	22.6	3.58

* Tuning method: HM = Harris and Mellichamp; EHH = Edgar, Heeb, and Hougen; ZN = Ziegler and Nichols.

TABLE 3. RESULTS FOR THIRD-ORDER PROCESS

Mode	Tuning Method	Process Model	K_c	τ_I	τ_D	M_r	ω_r	PM	GM
PI	HM	3	2.31	10.66	—	1.30	1.06	50.8	3.14
		3/2DT*	2.75	12.04	—	1.58	1.15	41.6	2.67
	ZN	3	3.60	3.02	—	4.39	1.25	14.8	1.52
PID	HM	3	5.38	7.94	1.98	1.30	3.04	45.2	—
		3/2DT*	5.80	2.90	0.725	1.30	1.77	45.4	—
	ZN	3	4.80	1.81	0.454	1.89	1.37	24.1	—

* Second-order with dead-time approximation of third-order process: $K_p = 1$, $\tau_d = 0.376$, $\tau_1 = 1.504$, $\tau_2 = 1.494$.

TABLE 4. RESULTS FOR FOURTH-ORDER PROCESS

Mode	Tuning Method	Process Model	K_c	τ_I	τ_D	M_r	ω_r	PM	GM
PI	HM	4	2.76	4.32	—	1.30	1.04	47.0	3.75
		4-pert.	2.73	3.90	—	1.33	1.03	45.6	3.70
		4/2DT*	2.77	4.60	—	1.28	1.04	47.8	3.78
	ZN	4	5.67	2.34	—	5.36	1.54	11.6	1.49
PID	HM	4	9.46	2.32	0.580	1.30	2.46	45.2	5.1
		4-pert.	9.52	2.59	0.647	1.27	2.76	46.3	4.81
		4/2DT*	9.20	2.32	0.581	1.28	2.38	45.9	5.23
	ZN	4	7.56	1.40	0.351	2.03	1.93	23.4	4.63

* Second-order with dead-time approximation of fourth-order process: $K_p = 1$, $\tau_d = 0.258$, $\tau_1 = 1.823$, $\tau_2 = 1.212$.

TABLE 5. RESULTS FOR SIXTH-ORDER PROCESS

Mode	Tuning Method	Process Model	K_c	τ_I	τ_D	M_r	ω_r	PM	GM
PI	HM	6	1.64	6.06	—	1.30	0.657	54.2	2.48
		6/2DT*	1.635	5.52	—	1.34	0.653	52.1	2.44
	ZN	6	2.22	5.04	—	2.34	0.751	30.7	1.75
PID	HM	6	3.17	4.18	1.05	1.30	1.110	48.5	2.97
		6/2DT*	2.96	3.71	0.928	1.28	0.962	48.0	3.23
	ZN	6	2.96	3.03	0.757	1.54	0.858	38.7	3.11

* Second order with dead-time approximation of sixth-order process: $K_p = 1$, $\tau_d = 0.894$, $\tau_1 = 1.831$, $\tau_2 = 1.845$.

case where the PM and GM of the ZN system are very low. The controllers tuned using the approximate model are satisfactory, although somewhat different from those tuned using the actual model. It should be noted that this process is the least like the second-order with dead-time approximation used tó represent the process. Table 4 shows the results for the fourth-order system. Three models were used for comparison in design of the controllers: the actual model, a fourth-order model with perturbed parameters (in error by ±10%), and a second-order with dead-time model. All three models gave similar controllers, and all are superior to the ZN controllers in terms of speed of response and/or the oscillatory nature of the resulting closed-loop systems. Table 5 shows the results for the sixth-order process. Both the actual and approximate models yield the desired results. Parameters differ by only about 10%.

Note that the IP used to obtain these results could also be used to meet other criteria. For example, it is sometimes desired to specify a certain overshoot or a certain decay ratio in the transient response. By approximating the closed-loop system as an underdamped second-order system, the damping coefficient necessary to obtain the desired overshoot or decay ratio can be determined and this in turn used to specify the desired resonant peak. This technique can be used fairly successfully. In the examples given, an M_r of 1.3 was chosen. This corresponds to a damping coefficient of 0.425, which corresponds to an overshoot of 22.9%. In the example shown in Figure 1 the overshoot is 22.5%.

In most of the examples, the results were not sensitive to the starting values chosen for the optimization program, the exception being starting values which themselves give an unstable (or nearly unstable) system. In that case, different starting values are chosen. Tuning of the PI controllers was found to be slightly more sensitive to the choice of starting values. The best approach seems to be to run the program a few times using different starting values and to select the most favorable final values.

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LITERATURE CITED

Bollinger, K. E., P. A. Cook, and P. S. Sandhu, "Synchronous Generator Controller Synthesis Using Moving Boundary Search Techniques with Frequency Domain Constraints," *IEEE Trans. Power Apparatus Syst.*, **PAS-98** 1497 (1979).

Box, M. J., "A New Method of Constrained Optimization and a Comparison with Other Methods," *Computer J.*, **8**, 42 (1965).

Chidambara, M. R., "Chemical Process Control—A New Technique for Adaptive Tuning of Controllers," *Intern. J. Control*, **12**, 1057 (1970).

Cohen, G. H., and G. A. Coon, "Theoretical Investigations of Retarded Control," *Trans. ASME*, **75**, 827 (1953).

Edgar, T. F., R. Heeb, and J. O. Hougen, "Computer-Aided Process Control System Design Using Interactive Graphics," *Computers Chem. Eng.*, **5**, 225 (1981).

Harriott, P., *Process Control*, McGraw-Hill, New York (1964).

Lopez, A. M., C. L. Smith, and P. W. Murrill, "An Advanced Tuning Method," *Brit. Chem. Eng.*, **14**, 1553 (1969).

Luenberger, D. G., *Introduction to Linear and Nonlinear Programming*, Addison-Wesley, Reading, Mass. (1973).

Yuwana, M., and D. E. Seborg, "A New Method for On-Line Controller Tuning," *AIChE J.*, **28**, 434 (1982).

Zakian, V., and U. Al-Naib, "Design of Dynamical and Control Systems by the Method of Inequalities," *Proc. IEE*, **120**, 1421 (1973).

Ziegler, J. G., and N. B. Nichols, "Optimum Settings for Automatic Controllers," *Trans. ASME*, **64**, 759 (1942).

Studies in Chemical Process Design and Synthesis

Part VII: Systematic Synthesis of Multipass Heat Exchanger Networks

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An important process design problem is the systematic synthesis of energy-optimum and minimum-cost networks of exchangers, heaters and/or coolers to transfer the excess energy from a set of hot streams to streams that require heating (cold streams). The specific problem statements, common simplifying assumptions, and systematic synthesis techniques along with a list of stream specifications and design data for example problems reported in the literature can be found in Part III (Nishida et al., 1977). Essentially all of the work reported thus far has been limited to the use of single-pass countercurrent shell-and-tube exchangers, heaters and coolers. In this paper, a simple approach to the systematic synthesis of energy-optimum and minimum-cost networks, which may utilize multipass exchangers, heaters and coolers, is proposed and demonstrated.

THERMAL DESIGN OF MINIMUM-COST MULTIPASS EXCHANGERS, HEATERS AND COOLERS

Multipass exchangers, heaters and coolers are often used in the process industries. Such equipment allows for a great deal of flexibility to a given heat-exchange process design. For example, multiple-shell passes may be used: (i) to improve the temperature difference between hot and cold streams for a given exchanger, in which the stream flows are not parallel or countercurrent; and (ii) to decrease the amount of floor space required for a given exchanger. Multiple-tube passes may be used: (i) to increase the fluid velocity in the tubes, thereby increasing the overall heat transfer coefficient; and (ii) to increase or decrease the available heat transfer area without increasing or decreasing the shell length. The numbers of shell and tube passes are limited by the maximum allowable pressure drop and by space considerations. Further, the number of tube passes is also limited by the amount of fluid passing through the tubes and by the maximum permissible shell diameter.

The basic equation for the thermal design (i.e., finding the heat transfer area) of multipass exchangers, heaters and coolers is (Bell, 1984):

$$A = \frac{Q}{U(\mathrm{MTD})} = \frac{Q}{UF_N(\mathrm{LMTD})} \quad (1)$$

Here, F_N is a correction factor that is so determined that when it is multiplied by the logarithmic mean temperature difference (LMTD) for a single-pass counter-current exchanger, the product F_N(LMTD) represents the true mean temperature difference

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